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SUMMARY OF EXPERIMENTAL ASTRONOMY LABORATORY WORK ON NASA GRANT NsG ~~364-62~~ 254-62

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SUMMARY OF EXPERIMENTAL ASTRONOMY LABORATORY WORK ON NASA GRANT ~~NsG 264-~~62

NSG-254

ABSTRACT

This report contains summaries of talks given by staff members of the M. I. T. Experimental Astronomy Laboratory at a meeting on 12 January 1965. The purpose of the meeting was to acquaint representatives from the various NASA Centers with the research being done under NASA Grant NsG 254-62. The topics were

1. Analytic studies on space navigation techniques.
2. Educational activities at the Experimental Astronomy Laboratory
3. Inertial gyros in space navigation systems
4. A sensitive low-level accelerometer.

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J. I. Kanter
C. Pontius

Lewis Research Center

H. Gould
W. Nieberding

Marshall Space Flight Center

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Prof. W. Hollister
Prof. J. Potter
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CHAPTER I

INTRODUCTION

This report contains summaries of talks given by staff members of the M. I. T. Experimental Astronomy Laboratory in a meeting at M. I. T. on 12 January 1965 The purpose of the meeting was to acquaint representatives from the various NASA Centers with the research being done under NASA Grant NsG 254-62.

The Experimental Astronomy Laboratory (EAL) is one of several faculty-directed laboratories in the Department of Aeronautics and Astronautics, which is headed by Professor C. S. Draper. The Laboratory is also affiliated with the M. I. T. Center for Space Research, an interdepartmental organization directed by Professor J. V Harrington EAL's staff consists of five faculty members, nine professional research engineers, and six graduate assistants. Professor W. Wrigley is EAL's acting director while the Laboratory's founder and director, Associate Professor W. R. Markey is on leave as Chief Scientist of the Air Force.

At the present time, about two-thirds of the Laboratory's work is supported by NASA funding and this work, outlined at the meeting, is on analytic studies of space-vehicle navigation and on basic physical properties of navigational instruments--particularly on experimental investigations of exotic inertial-sensors. Until September 1964, all of the research outlined in this report was supported by NASA Grant NsG 254-62; now the low-level accelerometer research (Chapter IX) is supported by a separate NASA grant, and we expect the studies on inertial gyros for space navigation systems (Chapters VII and VIII) to be supported by another NASA grant

The research outlined in this report has been fruitful in providing thesis topics for graduate students. In addition to regular laboratory reports, the Bibliography, Chapter X, lists four doctoral theses, one engineer's thesis, and nine master's theses. The Laboratory's educational program continues to support graduate student thesis-research and to provide material for classroom lectures. Assistant Professor W. Hollister outlines these academic activities in Chapter III of this report.

The Experimental Astronomy Laboratory is divided into two groups--the Astroguidance group and the Experimental group. The Astroguidance group is led by Dr. Robert Stern and their work is described in Chapters II-VI. The Experimental group is led by B. Blood and their work is outlined in Chapters VII-IX.

CHAPTER II

RESUME OF ASTROGUIDANCE GROUP ACTIVITY

Robert G. Stern

The Astroguidance group's activity can be divided into three main categories: 1. mission planning, 2. midcourse guidance, and 3. statistical theory and determination of strategies. Each of these activities is under the supervision of one of the group's staff engineers: Professor Hollister for mission planning, Dr. Stern for midcourse guidance, and Professor Potter for statistical theory. Professors Hollister and Potter describe their projects in greater detail in later chapters of this report. A detailed analysis of the midcourse guidance research is presented in Mr. Slater's report. Mr. McDonald reports on the development of a trajectory program which will be useful as a tool in all the phases of the group's research activity.

The emphasis in much of the work being done is on an analytic rather than a numerical approach to the problems of interplanetary flight. Relatively simple mathematical models are devised in order to obtain a better physical understanding of the problem. Numerical studies are made to determine the relative accuracy of the simple models with respect to a more complex "standard" model.

In interplanetary midcourse guidance there are two basic simplifications that can be tested. The first involves neglecting, or treating in a relatively simple fashion, the effects of all perturbations on the classic two-body problem. The second basic simplification involves linearization of the model based on the assumption that all variations of the actual trajectory from a known trajectory are small.

These two types of simplifications may be applied to either explicit or implicit guidance systems. Explicit systems are those which do not rely on a precomputed trajectory but do require on-board numerical integration. Implicit systems, on the other hand, require no on-board numerical integration but do rely on a pre-computed reference trajectory.

Numerical results, as shown in Mr. Slater's report, indicate that virtually all the simple models, using either explicit or implicit techniques, can be made to yield acceptably accurate midcourse guidance corrections. The choice of midcourse guidance system for a particular mission can be based on such considerations as hardware weight, complexity, and reliability, without being greatly influenced by the relatively inconsequential amount of fuel expended or position accuracy at the destination.

As an extension to the work in midcourse guidance, a study is now being initiated in simplified midcourse navigation. The aim will be to devise techniques of position determination and orbit determination from redundant data. These techniques do not necessitate the use of digital computation. An attempt will be made to devise simple models in order to achieve the objective.

The academic links of the group's activity are described in some detail by Professor Hollister. The prospects are that there will be even greater academic participation by the group, in the form of sponsored doctoral and master's thesis research, during the coming year.

The primary operational problem faced by the group is the lack of adequate digital computer facilities. Although much has been done during the past few months to alleviate this problem, much more remains to be done if the group is to perform in accordance with its potential.

CHAPTER III

MISSION PLANNING AND ACADEMIC ASPECTS OF EAL ACTIVITY

Walter Hollister

Mission Planning

The advantages of passing Venus during a round-trip, stop-over mission to Mars are now becoming well known. The general idea of passing Venus enroute to Mars was first suggested by Crocco. It was studied here in 1962 and reported in EAL Report TE-4 in 1963. Those results were based on the utilization of a small velocity increment near Venus. Recent work by Sohn of STL has considered only pure flybys of Venus without thrust. A natural question arose as to the magnitude of the saving achieved by the use of a velocity increment over the pure flyby of Venus. The specification of the launch date at Earth and the arrival date at Mars is sufficient to determine a direct transfer to Mars or a flyby of Venus enroute to Mars without thrust near Venus. Computation of a thrusted flyby of Venus for the given dates at Earth and Mars requires a complicated optimization. For each possible date at Venus it is necessary to compute the optimum place near Venus to make the velocity change. The work done here confirmed results reported earlier by Gobetz of United Aircraft and provided additional insight into the problem through a different approach. This approach included the constraint imposed by the planet in a simple manner. It was found that the common peripoint correction is within a few percent of the true optimum for practical trajectories. Using the results of the optimum flyby study, the best flyby with thrust was computed for most practical dates between 1970 and 1990. It was found that the transfer using thrust during the flyby of Venus is usually better than one which does not use thrust, but the saving is small, around a few hundred feet per second. This is because the optimum date for the flyby of Venus with thrust is always close to the date for a pure flyby of Venus. It was also found that there are some dates when a thrusted flyby of Venus is more economical than direct flight, and at the same time a flyby of Venus without thrust is not possible without striking the planet's surface. When this situation occurs there are usually neighboring dates for which flights of the other two types are more economical. Consequently, the use of thrust during the flyby of Venus does offer savings, but from a practical point of view they do not appear significant. The conclusion applies only to the specific type of Earth-to-Mars transfer for a round-trip, stopover mission which was studied. Because the thrusted flyby always has a potential for making a saving over the pure flyby, it is desirable to investigate the magnitude of such potential. The method of analysis developed is applicable to any interplanetary flyby trajectory. The results of this study will be documented in a report now being written.

Academic Aspects of EAL Activity

One aspect of the laboratory's activity is to encourage graduate student participation in the research which is being carried out and also ensure that the results of the research are disseminated to interested students throughout the Institute. Evidence of graduate student participation is given by the number of theses done in connection with EAL research. In the spring of 1964 there were three master's theses completed by Holbrow, Gielow and Munnell. These were all concerned with techniques for simplified interplanetary guidance. The significant results will be included in a comprehensive report on simplified guidance now being prepared. The theses are currently available through the M. I. T. Microreproduction Laboratory. Two other master's theses by Chapman and Ezekiel were also completed in June 1964. Their work was on a sensitive low-level accelerometer. A doctoral thesis by LCDR Mitchell on guidance for low-thrust vehicles was completed and has been circulated as EAL Report TE-8. This term there are two additional master's theses and one engineering thesis being completed. A master's thesis by Captain Ruth is titled, "Analysis of an Interplanetary Line-of-Sight Guidance Technique." A master's thesis by McFarland is titled, "A Digital Computer Study of Interplanetary Guidance by Observation of Star Occultations." An engineering theses by Carlson is titled, "Correlation of Interplanetary Geometry with Propulsion Requirements for Optimal Low-Thrust Missions." Each of these theses represents a significant contribution.

In an attempt to bring the results of the laboratory's research back to the students, a new graduate course has been introduced called "Special Problems in Interplanetary Flight." The course was taught last Spring by Professor Hollister, Dr. Stern, and LCDR Mitchell to about fifteen students. It was well received and served as the incentive for many of the graduate theses which have been undertaken. A few of the original students have come to work with the analytic group. It is intended that the course will be offered every year and that current research will keep the material alive and up to date.

CHAPTER IV

SIMPLIFIED GUIDANCE TECHNIQUES

Gary Slater

The purpose of this investigation is to examine various methods that have been suggested as simplifications to the mid-course guidance problem. The methods investigated are all FTA (fixed-time-of-arrival) and are designed for use in self-contained guidance systems. The guidance techniques can be broken into two broad categories. These are termed "implicit" and "explicit", respectively. By definition, an implicit system is one which utilizes a pre-stored reference trajectory to generate needed information and does not require an on-board numerical integration of the equations of motion. The implicit techniques have been studied by various investigators at the Experimental Astronomy Laboratory (e. g. Stern(1), Munnel(2), or Gielow(3)). Conversely, the explicit technique requires no reference trajectory but does use a numerical integration of present calculation of a velocity correction. Explicit techniques have been examined by several investigators,for example Battin(4) of Instrumentation Laboratory.

(1) Stern, Robert G., "Interplanetary Midcourse Guidance Analysis", EAL Report TE-5, May 1963.

(2) Munnel, Thomas C., "An Analysis of the Application of Two-Body Linear Guidance to Space Flight", S. M. Thesis, M. I. T., August 1964.

(3) Gielow, Robert L., "An Evaluation of the Two Body Non-Linear Guidance Technique", S. M. Thesis, M. I. T., May 1964

(4) Battin, Richard W., "Explicit and Unified Methods of Spacecraft Guidance Applied to a Lunar Mission", XVth International Astronautical Congress, Warsaw, Poland, September 1964.

The six methods will now be explained:

1. Linear Guidance

In linear guidance we utilize a precomputed reference and linearize about this reference in terms of the position and velocity deviatons from the nominal. In the many-body case the matrix differential equations are integrated prior to launch and stored on board the spacecraft. To calculate the velocity correction it is necessary only to estimate the state deviation and perform a matrix multiplication.

2. Phantom Target

The phantom target technique is an implicit method which utilizes the non-linear equations of two-body motions but makes simplifying assumptions about the disturbing accelerations. The assumption made is that the integrated effect of the accelerations is constant for trajectories lying near the reference. The difference between the actual destination point and the destination as predicted by the osculating conic on the reference is a measure of this effect and hence to reach the target we make a two-body transfer from our present position to the destination point predicted by the osculating conic.

3. Position Offset

It is possible to use the same concept in an explicit way by integrating the present position and velocity and using this as a reference trajectory. We find the position offset on this trajectory and assume this difference will remain constant when we aim to hit the target.

4. Two Body Linear

The two-body linear method is the same as the phantom target method except that it attempts to null the miss on the osculating conic in a linear way. This system has the advantage that the differential equations can be integrated analytically and no prior numerical integration of the transition matrices is required.

5. Velocity Offset

The velocity offset is utilized in an explicit manner by first integrating the true position and velocity to find the destination point if there is no velocity correction. Then a two body conic is passed through these two points. The velocity difference between these two trajectories can be regarded as the velocity needed to offset the effect of the perturbing accelerations. If this difference is regarded as constant we calculate the velocity needed for a two-body trajectory to the target and then add this velocity offset to it. By our assumption this gives us the correct velocity to reach the target.

6. Line-of-Sight

The line-of-sight method is a relatively crude technique which is used here only to point up the extent to which we can simplify a guidance system. In this method the gravity forces are neglected entirely and straight line motion is assumed. By prestoring reference heading and range values a velocity correction can be made whenever the heading angle to the targe differs by a specified tolerance. Though not conservative with fuel, it is possible by this method to reduce significantly the miss at the target point

To test the accuracy of the methods, two reference trajectories were given initial launch perturbations and then a velocity correction was made. The results of a first correction at twenty days after launch are shown in the following tables.

The symbols used in the tables are:

$\lvert \underline{C} \rvert$	Magnitude of the velocity correction in meters/second
T_C	Time of correction measured from launch in days
$\lvert \delta_r \rvert$	Magnitude of miss vector at the target (no velocity correction) in kilometers

$|\delta_r|^+$ Magnitude of miss vector at the target (with a velocity correction) in kilometers

ΔV Perturbation in launch velocity either in total ΔV (e. g., 1 m/s) or along each coordinate direction (e. g., 1, 1, 1 m/s).

$\Delta V = 1$ m/s $\quad T_C = 20$ days $\quad |\underline{\delta r}|^- = 150{,}000$ km

METHOD	$\lvert\underline{C}\rvert$	$\lvert\underline{\delta r}\rvert^+$
<u>IMPLICIT</u>		
Phantom Target	15.86	426
2-B Linear	15.87	382
<u>EXPLICIT</u>		
Velocity Offset	12.73	17,300
Velocity Offset - Aim at Sph. Inf.	16.04	1,533 (404)*
Position Offset	12.73	17,250
Position Offset - Aim at Sph. Inf.	16.04	1,436 (340)*

Line-of-Sight - 6 Corrections $\quad \Sigma|\underline{C}| = 91$ m/s

Miss = 700 km

*Number in parenthesis indicates the miss at the actual aim point near the sphere of influence of the planet.

Table 1 Venus 101 day trajectory.

$\Delta V = 1, 1, 1$ m/s $\quad T_C = 20$ days $\quad |\underline{\delta r}|^- = 330,000$ km

METHOD	$\lvert\underline{C}\rvert$	$\lvert\underline{\delta r}\rvert^+$
IMPLICIT		
Phantom Target	5.59	3,400
Linear (Many Body)	5.61	3,272
2 - B Linear	5.33	7,980
EXPLICIT		
Velocity Offset	5.10	12,012
Velocity Offset - Aim at Sph. Inf.	5.64	4,570 (2895)*
Position Offset	5.07	13,005
Position Offset - Aim at Sph. Inf.	5.59	3,630 (2099)*

Line-of-Sight - 15 Corrections $\quad \Sigma|\underline{C}| = 299$ m/s

Miss = 450 km

*Number in parenthesis indicates the miss at the actual aim point near the sphere of influence of the planet.

Table 1 (Cont.) Mars 258 day trajectory.

ΔV = 10, 10, 10 m/s $\quad T_C$ = 20 days $\quad |\underline{\delta_r}|^-$ = 3,340,000 km

METHOD	$\lvert\underline{C}\rvert$	$\lvert\underline{\delta_r}\rvert^+$
IMPLICIT		
Phantom Target	55.90	15,035
Linear (Many Body)	56.04	4,607
2 - B Linear	53.22	92,415
EXPLICIT		
Velocity Offset	55.68	7,377
Velocity Offset - Aim at Sph. Inf.	55.95	19,940 (14,240)*
Position Offset	55.30	17,640
Position Offset - Aim at Sph. Inf.	55.66	30,406 (22,421)*

*Number in parenthesis indicates the miss at the actual aim point near the sphere of influence of the planet.

Table 1 (Cont.) Mars 258 day trajectory.

CHAPTER V

OTHER SPACE NAVIGATION AND GUIDANCE PROBLEMS

James Potter

In this section, work in fairly mathematical areas of guidance theory will be discussed. The areas covered are: optimum nonlinear midcourse correction strategies for an interplanetary flight with and without navigational uncertainties; the solution of a matrix equation arising in the optimal control of a linear plant and statistical filtering theory, and finally a linearized midcourse guidance law for a low thrust interplanetary vehicle based on the second variation method in the calculus of variations.

1. Optimization of Midcourse Velocity Corrections

In this study it is assumed that the miss due to injection errors can be predicted exactly (perfect navigation) and that the effects of velocity corrections may be calculated using the equations of motion linearized about the reference trajectory of the spaceship. Given the miss vector at the target that would occur without velocity corrections, the problem is to determine the set of velocity corrections which will eliminate this miss using the least fuel (minimum total Δv). It was found that the number of velocity corrections required by the optimum strategy is never larger than the number of constraints at the target. Thus if both the vehicle's position and velocity are to attain fixed values (rendezvous) at a fixed arrival time, there are six constraints and as many as six velocity corrections may be needed to minimize fuel. If only position is to be matched at a fixed time of arrival, there are three constraints and at most three corrections. Finally, for variable time of arrival with only target position to be matched (position VTA) there are two constraints at the target and at most two corrections are needed to minimize fuel. Since the number of constraints at the target is only an upper bound on the number of corrections, there was some question concerning whether two corrections are ever better than one with a reasonable reference trajectory in the "position VTA" case.

A geometrical construction involving convex sets was developed to determine the optimum velocity correction strategy. In the two constraint "position VTA" case, this construction may be carried out graphically and cases when two corrections are better than one may be easily found on the graph if they exist. A number of two body elliptic reference trajectories have been studied for "position VTA" guidance. A fairly dramatic example is provided by a Hohman transfer with an eccentricity of 0.95. When flying from perigee to apogee, a fifty percent saving in fuel can be achieved by using two velocity corrections rather than the best single correction for some directions of miss. However, on the return trip, flying from apogee to perigee, no saving results from making multiple velocity corrections The possible saving through multiple corrections seems to decrease with the eccentricity of the reference trajectory, although a small saving is possible even in some circular orbit cases.

In some cases it is also possible to formulate a simple near optimum midcourse guidance law using the geometrical construction employed for finding the optimum strategy.

A report covering this work is being prepared by Dr. Stern and myself.

2. Matrix Quadratic Equations

A method has been developed for finding all of the matrices x which satisfy the matrix quadratic equation

$$XAX + BX + XB^T + C = 0$$

where A, B and C are given coefficient matrices. Matrix quadratic equations arise in determining the optimum feedback gains for controlling a plant described by linear differential equations with constant coefficients where the cost function to be minimized is the integral of a quadratic form in the deviation of the plant output from the desired value and the control effort. The spectrum factorization step in designing a Wiener filter to extract a signal from noise can be replaced by solving a matrix quadratic equation. However, this is not necessarily an advantage for computation since the spectrum factorization and matrix quadratic methods are about equally complex.

3. Optimum Midcourse Guidance Strategy in the Presence of Navigation Uncertainties

If midcourse navigation is carried out by means of optical sightings, the uncertainty in the predicted miss at the target decreases substantially as more sightings are made and the target is approached. If navigation uncertainties are neglected, early velocity corrections tend to use less fuel since they have a longer time to take effect. However, if navigation uncertainties are considered, early velocity corrections designed to remove all of the estimated miss at the target are inefficient since some of the estimated miss is due to navigation errors and does not represent a real physical miss. Thus, it appears that early velocity corrections should not correct all of the estimated miss. Breakwell has studied general linear strategies for a one dimensional midcourse guidance problem with navigation uncertainties. He made six or eight corrections, correcting a certain percentage of the estimated miss at each correction time. These percentages were calculated using the calculus of variations. Breakwell found that indeed one should undercorrect at the early correction times. In the present work general nonlinear strategies have been found for one and two dimensional problems where only two velocity corrections are to be made.

In order to minimize the dispersion at the target, the second correction in a two correction strategy should remove all of the estimated miss. For the first correction in the one dimensional case there is a simple threshold effect. If the estimated miss at the first correction time is less than the threshold, no velocity correction is made. If the estimated miss is larger than the threshold value, only the amount of miss above the threshold is corrected.

In the two dimensional case, which corresponds to "position VTA" guidance, the optimum strategy calculations involve elliptic integrals. There is again a threshold for the first correction, this time an approximately elliptical region. If the estimated miss vector lies inside of this region, no correction is made. However, if the miss vector lies outside of the threshold region the miss corrected is no longer simply the portion of the miss vector outside of the threshold region.

It is planned to work out numerical examples and compare the optimum nonlinear strategy with simpler guidance laws. Future work will also include trying to find fairly simple formulas for the optimum strategy when more than two midcourse corrections are to be made.

4. Guidance for a Low Thrust Interplanetary Vehicle

It is intended to carry out a digital simulation of a midcourse guidance system for a low thrust vehicle employing the second variation technique of the calculus of variations. The EAL trajectory computer program will form the core of the simulation and as soon as it is completely checked out, work can proceed on this project.

CHAPTER VI

SPECIAL PURPOSE SPACE TRAJECTORY PROGRAM FOR GUIDANCE STUDIES AND SUNBLAZER VEHICLE TRAJECTORY STUDIES

William McDonald

Trajectory Program

A program has been written to compute interplanetary trajectories in a many-body gravitational field for use in guidance studies in the Astroguidance Group. The program serves as an evaluation standard in these studies, i.e., it is used to search for and establish nominal trajectories and to compute the non-linear effects of deviations and corrections along the nominal trajectory. A number of trajectory runs are required in the evaluation of a guidance technique, so that economy of operation is a primary requirement for the program. Also, to facilitate automatic processing at the M.I.T. Computation Center, the program operates with a standard monitor system and does not require input data tapes (e.g., tape-stored planet ephemerides). Because a number of different kinds of studies are anticipated, the program is written in FORTRAN to permit easy modification by engineering programmers.

The program computes both the state vector (from the many-body equations of motion) and the state transition matrix (from the state variational equations). Trajectories can be computed backward in time, as well as forward, to facilitate economical computation of transition matrices at several points along a trajectory. The forward-backward capability permits a unique "closed loop" computational accuracy check in the program.

The Encke method of orbit integration and Nystrϕm's method of numerical integration are used. The planets Venus, Earth, Mars, Jupiter, and Saturn are included, but the planet ephemerides are computed from osculating conics approximating the planet orbits. Since the program is used for comparative studies of guidance techniques, the imprecise ephemeris does not penalize conclusions reached about relative merits of the techniques.

The state transition matrix is used to implement an iterative trajectory-search capability which exhibits good efficiency since the correct state transition matrix is computed in each iteration. Three printout options are provided, including one for printout at up to 20 time points per trajectory specifiable by means of input data cards.

Single precision computation is used throughout the program, and a very significant problem has been that the conic equations for computation of the conic state variables in the Encke method are very sensitive to computational imprecision in the case of a hyperbolic orbit. A reformulation of Kepler's hyperbolic equation for high precision computation has been carried out. The new formulation achieves a significant improvement in computational accuracy. It was not found possible to compute the hyperbolic conic state variables to single-precision accuracy with either the standard conic formulation or Battin's universal formulation, but the new formulation does yield the conic state variables to single-precision accuracy in the case tested (a close pass of Mars with peripoint 100 km above the planet surface).

A report on capabilities and methods of the program is in preparation and should be available by March 1, 1965.

SUNBLAZER Trajectory Studies

SUNBLAZER is a NASA-sponsored Sun probe vehicle now being developed at the M. I. T. Center for Space Research. The vehicle will weigh in the neighborhood of 10-20 lbs., contain two high-frequency radio transmitters, and will be launched to study electromagnetic characteristics of the corona of the Sun. The booster currently envisioned is an unguided rocket of a configuration similar to the Scout. The planned trajectory has a perihelion of about 0.5 a.u. and achieves a superior conjunction with earth at the same time as perihelion.

The efforts of the Astroguidance Group on behalf of SUN-BLAZER have been funded by that project and have been conducted to obtain some preliminary design data to establish spacecraft configuration requirements. These data include orbital parameters, orbit sensitivity coefficients, and Earth-probe-Sun tracking angles.

The computer programs and computational methods were conveniently available to perform these studies and the studies furnished an opportunity for the Experimental Astronomy Laboratory to lend support to another NASA project in a generally economical manner.

CHAPTER VII

INERTIAL GYROSCOPES IN SPACE NAVIGATION SYSTEMS

Joel Searcy

In recent years, much effort has been devoted to the study of drift in the floated integrating gyroscope. These studies have resulted in lists of possible drift sources, a multitude of tests to isolate specific causes of drift, and several schemes for correcting predictable drift components. Most of the effort devoted to the gyro drift problem to date, has been motivated by either a need for data leading to a design improvement in the instrument or an adequate acceptance-testing procedure for production units. The designer of navigation and guidance systems has tended to leave the final trimmings of system parameters until a prototype was built. This procedure is often the most efficient and desirable means of optimizing a system design, but it is not always possible. The design of systems for operation in space requires an entirely new design philosophy. Firstly, optimal system design becomes much more important in space where weight is costly and where small errors tend to have large effects in time. Secondly, the system designer can no longer rely on "screwdriver optimization" of a prototype, because he does not have available the environment in which the system must ultimately operate. It is therefore becoming increasingly important that the system designer have data on the statistical nature of component uncertainties in order that optimum control tactics may be formulated. Powerful theoretical tools have been provided by the work of Wiener, Kalman, Battin and many others, but a basic assumption common to all statistical optimization schemes is that the statistics of the uncertainty inputs are known. These considerations then lead to the following observations.

1. Accurate analytical statistical models for the uncertainties in inertial components operating in the ground test environment should be obtained.

2. The effects of the space environment, particularly the zero-g environment, on the drift statistics should be evaluated "in situ".

By means of a careful choice of models and characteristic parameters, the results of these two measurements would provide a further insight into the physical sources of drift, a possible means of extrapolating performance in the ground environment into the space environment, and an opportunity to evaluate any unknown or unpredictable effects which might originate in the space environment.

In the long run, measurements of this type in space will probably be a part of the mission of a manned space station. This would involve a relatively elaborate and flexible test facility capable of testing a wide variety of navigation sensors, both inertial and optical. The efficient design of such a test facility with respect to both hardware and procedures would benefit greatly from an initial unmanned experiment to evaluate the performance of a floated integrating gyroscope in the space environment.

Much of the current literature on the use of gyroscopes in space emphasizes the increased uncertainty levels tolerable in space navigation applications due to the use of optical sensors. This reasoning has been interpreted by some as meaning that there is no need to obtain in advance the qualitative data on gyro performance mentioned above. The fallacy in this interpretation lies in the fact that we must seek systems that are optimum with respect to cost, weight and reliability as well as to performance. The designer of a space navigation system must then ask the question, "What is the cheapest, lightest and most reliable gyro that will meet the performance requirements of this particular mission."

At any specified level of performance, the designer must still have available an accurate statistical model for component uncertainties in order to make use of optimum design theory.

The fact that state-of-the-art components are not required for a particular mission then means that the designer has a larger set of components from which to choose and hence must be provided with the data necessary to make an optimum choice. We therefore see that the increased uncertainty levels tolerable in space results in a greater need for performance data in the space environment rather than making such data unnecessary.

CHAPTER VIII

APPLICATION OF STATISTICAL THEORY TO TESTING OF GYROSCOPES

John Hendrickson

The aim of most analyses of practical instruments is the development of a tool which will be both useful and accurate in predicting performance. In the present case, what is desired is a mathematical model, developed from actual data, which will determine the performance of a gyroscope under a given set of conditions. Since Fourier analysis of gyro test data inherently contains errors due to gyro drift, other methods must replace the data evaluation techniques now being used. In this way these instruments will become more useful to the designer of navigation and guidance systems.

Some of the early work along these lines concerns itself with the application of correlation techniques. The correlation function being defined as the limit of the average of the product of two functions, one of which is displaced with respect to the other. The limit being taken as the interval, over which the average is made, goes to infinity. For auto-correlation, the two multiplied functions are identical and for crosscorrelation, they are different. The characteristics of the correlation functions are such that if there are periodic components common to both functions, the correlation function will contain the same periodic components. If no common periodic components are present, the correlation function will approach, for large displacements, the square of the average value. Thus, correlation functions expose common components and the randomness of sets of functions.

Hammon[1], assuming gyro drift rate as a stationary random process, examined the representation of the gyro drift rate autocorrelation function as

$$\phi_{xx}(\tau) = A^2 e^{-c|\tau|}$$

where

$\phi_{xx}(\tau)$ = autocorrelation function for stationary random gyro drift rate,

A^2 = mean-square error value,

$\frac{1}{c}$ = correlation time.

Gyro drift rate being defined as the unpredictable and therefore uncorrectable error rate in gyro performance. He justified this model rather loosely, nevertheless it was used to derive means for predicting on a statistical basis, the random drift rate and angle of initially uncorrected and corrected free gyros.

The results yielded expressions for the answers to such questions as:

1) For an uncorrected gyro, how rapidly does the mean-square gyro drift angle build up after uncaging, when the mean drift rate is zero?

2) After a given free drift period, how does the drift angle value for a gyro drifting randomly at a given drift rate compare to the drift angle value due to a constant drift rate of the same magnitude?

3) After drift rate has been corrected instantaneously to zero, how rapidly does it again build up and to what probable ultimate value?

4) How long must a calibration run be to insure removal of, say, 95 percent of the mean drift rate?

5) For a short use time, what is the optimum averaging interval required to minimize the total error angle buildup?

The answers are not important in this discussion; however, the questions serve to indicate the use to which mathematical models may be put.

Dushman[2] applied this model one step further by making comparisons with data from actual units. Also included in the model was a mean-square term, namely

$$\phi_{xx}(\tau) = \sigma^2 e^{-\beta|\tau|} + m^2$$

where

$\phi_{xx}(\tau)$ = autocorrelation function,

σ^2 = variance of the process,

m = mean of the process,

$\frac{1}{\beta}$ = correlation time.

This model deviates from actual gyro drift data when the ensemble mean-square error of the drift rate is computed. A proposal of a second model which included a random walk along with the exponential type of random drift led to a better depiction of the mean-square rate. It was then concluded that the second model was the better one for the specific unit evaluated; however, it appeared that either of these models with a suitable change of parameter values might be applicable to some other unit.

Weinstock[3], using a simplified random-walk model, i. e., constant changes of +1, -1, 0 over a unit of time interval, each of equal probability, demonstrated the application of statistical analysis to the evaluation of stationary random gyro drift data. The application of these methods to actual data led to the questioning of the validity of the stationary random rate model. The autocorrelation functions do not agree with the previous exponential model and it was shown that the case of the bounded walk model would yield the exponential correlation function. This would indicate that the data used is not a true random sample in the sense that these data have drift angles and drift rates below some arbitrary level. The poorer performances have been unconsciously weeded-out. When these tests with poorer results were being performed, they were either halted to determine the causes or generally unreported. This effectively results in a boundary limit on the resulting data. When a statistically random sample is used, poor correlation exists between the model and the actual unit.

The results of the random sample indicated that the incremental change in drift rate, or drift acceleration, might be useful as an indicator of gyro performance. In later work[4], it was shown that the drift acceleration for certain types of gyro tests, is a grossly conservative performance indicator.

The conclusions to be drawn from the present position of the statistical approach to gyro evaluation is that drift rate is not to be treated as a statistically independent random process for the length of time involved in testing, and therefore any models proposed will have to be more sophisticated and more complex than those that have already been examined.

The efforts of the Experimental Astronomy Laboratory to the present have been along the more mundane lines of accumulating, designing and constructing equipment necessary for the testing of some gyroscopes now on hand. The initial test efforts will be thermal environment tests and will serve to indicate the relative merits of the equipment as well as indicate drift rate and thermal environment relationships.

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CHAPTER IX

THE LLAMA PROGRAM

Shaoul Ezekiel

Introduction

The objectives of the LLAMA program are:

1. To investigate on a theoretical as well as an experimental level, new techniques for the construction of compatible elements for a very sensitive accelerometer,

2. To construct a simple single axis linear accelerometer, incorporating the concepts evolved in the first part of the study, so as to demonstrate feasibility and to allow realistic performance evaluation.

The basic concept of the LLAMA program was outlined in a previous progress report PR-1, and in Ref. 1, 2, 3; a brief summary only will be given here. Basically, (See Fig. 1) a single axis linear accelerometer is being constructed using a proof mass consisting of a small permanent magnet supported by magnetic forces inside a superconducting cylinder. Coaxial coils on either side of the magnet, inside the cylinder, are used to compensate the axial instability of the magnet and also to provide the restoring force to keep the proof mass at null. A sensitive displacement detector is used to feed the restoring coils with control information. Calibration below 10^{-6}g is to be effected by controlled radiation pressure.

Evaluation of Suspension

A theoretical analysis has been made of the field distribution inside the suspension. The analysis indicated that the axial force inside the superconducting tube is given by

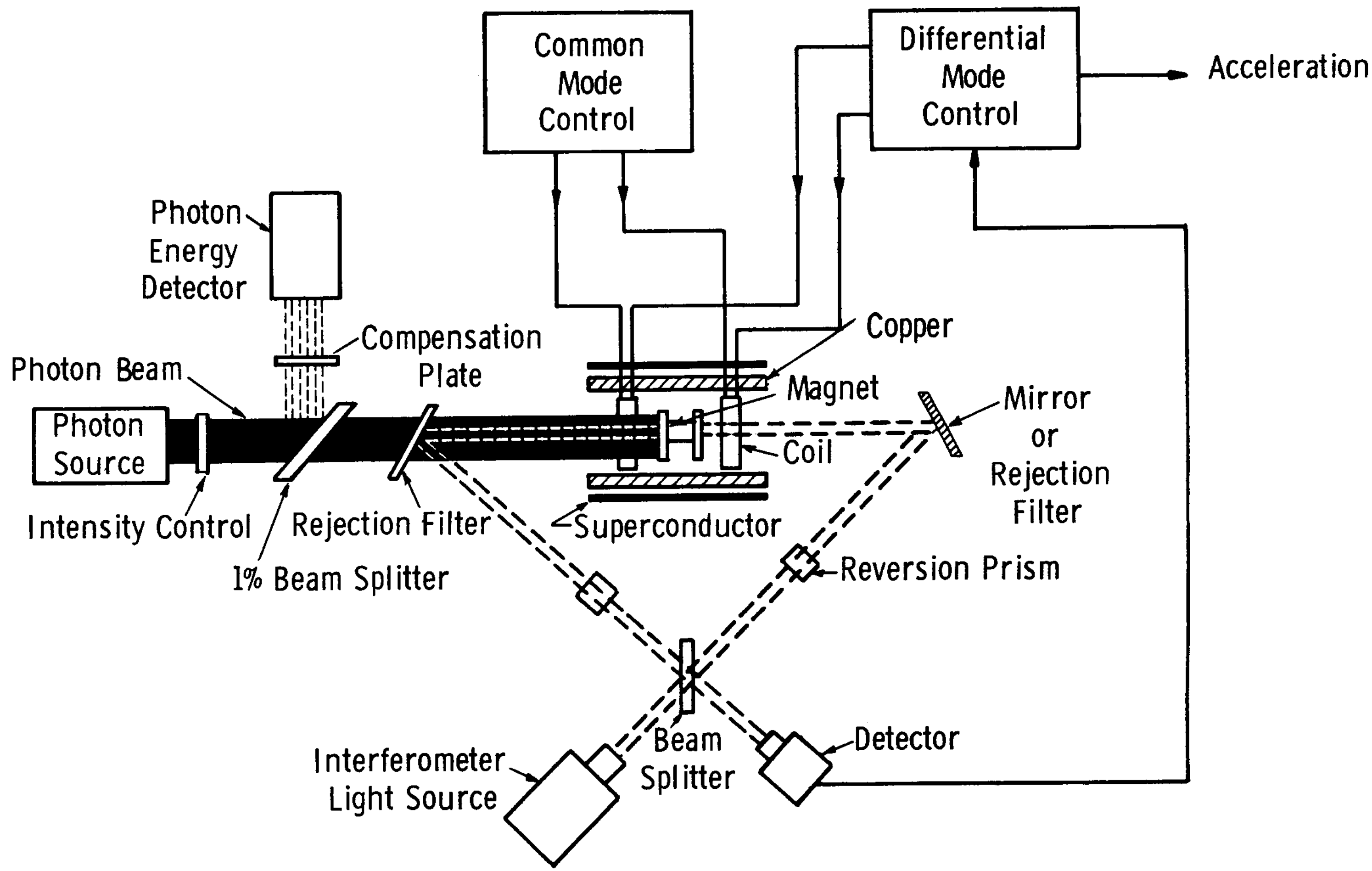

Figure 1 The Llama System

$$F_x = \alpha \sinh 2cx \tag{1}$$

where,

F_x = force on magnet at axial displacement x from null.

α = constant - depends on geometry of tube and magnet and on the pole strength of the magnet.

c = constant - depends on the radius of the tube.

The force on the magnet due to the coils is found to be,

$$F_{coils} = \frac{-\beta}{2}\left[(I + \Delta I)\, e^{cx} - (I - \Delta I)\, e^{-cx}\right]$$

$$= -\beta \; I \sinh cx - \beta \; \Delta I \cosh cox \tag{2}$$

where F_{coils} = force on magnet due to coils when magnet is at displacement x from null.

β = constant - depends on the geometry of the coils and the magnet.

c = constant as in Eq. (1).

I = standing current in each coils.

$2\Delta I$ = differential current in coils.

A sketch of the uncompensated axial suspension force and the compensating effect of the coils for small displacements from null is shown in Figure 2. For a suitably chosen current in the coils a small region exists around null where the magnet is axially stable; outside this region the suspension is unstable. The length of this stable region (taken as the distance between

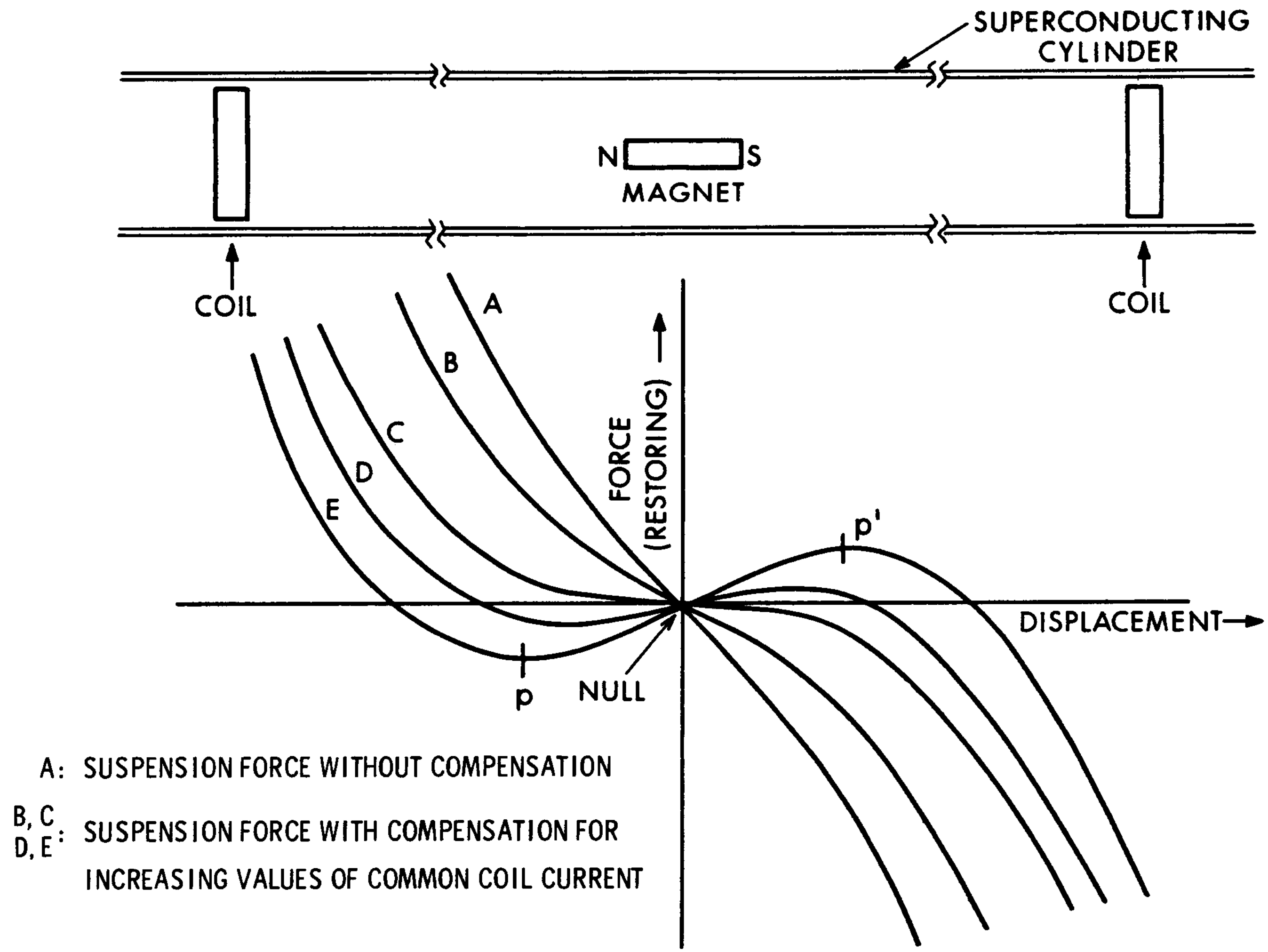

Figure 2 Axial Force on Magnet Close to Null

the peaks e.g. p-p' in Fig. 2) corresponding to a desired spring constant at null is shown in Figure 3. The smaller the desired spring constant, the shorter the stable region and hence the more sensitive the displacement detector has to be. One way of lengthening the stable region for a given spring constant is to reduce the inherent suspension spring constant as illustrated by the dotted curves in Figure 3.

In the absence of the interferometer (see below) an optical displacement detector using a spot occultation technique was constructed and attached to the dewar. The linear range of this detector was approximately 1 mm.

The effective spring constant in a 1mm region around null was determined by measuring the natural frequency of oscillation of the magnet. Figure 4 shows a plot of spring constant as a function of coil current. By extrapolation, the inherent suspension spring constant is found to be approximately -15 dynes/cm. Because of the general vibrations of the surroundings, the lowest value of spring constant measured was 2 dynes/cm.

Performance of Preliminary Accelerometer

By using the output of the spot occultation detector to control the differential current in the restoring coils, the closed loop behavior of this rather crude version of the accelerometer was investigated.

The overall block diagram for the accelerometer is shown in Figure 5. If the displacements are small the elements in the diagram can be linearized and the loop is seen to be unstable. This observation was substantiated by experiment and the loop was stabilized with the aid of some lead compensation. However, just before the loop did stabilize, an oscillation

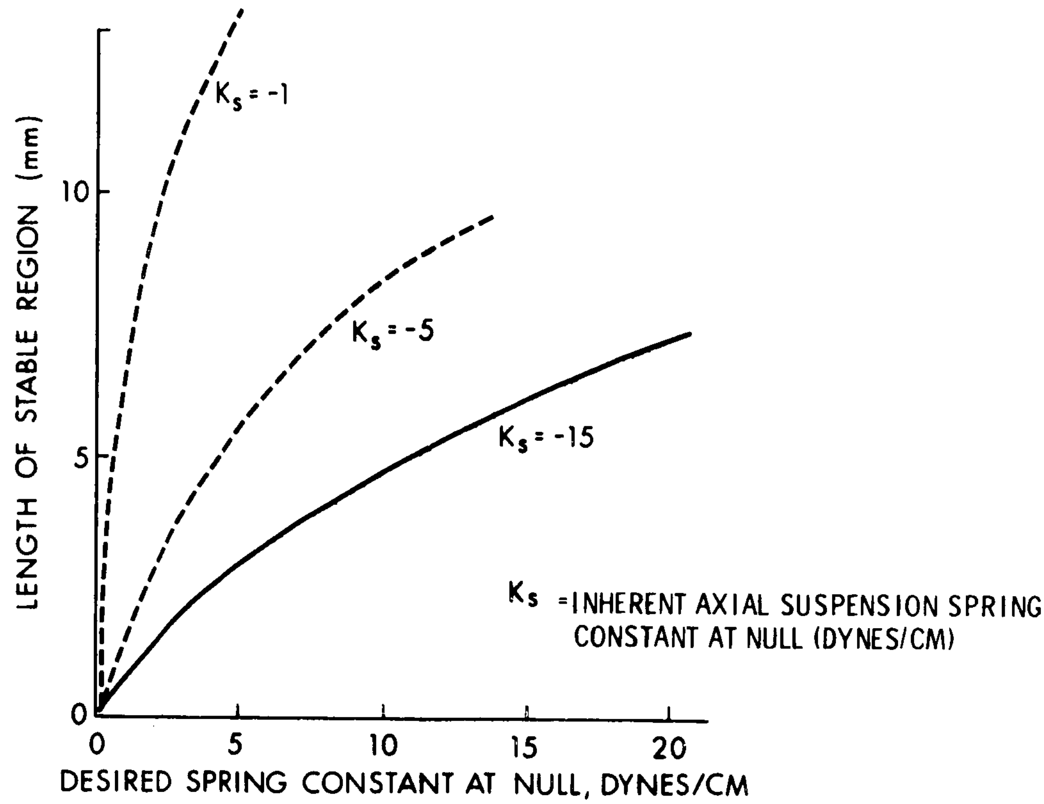

Figure 3 Length of Stable Region vs Desired Spring Constant at Null

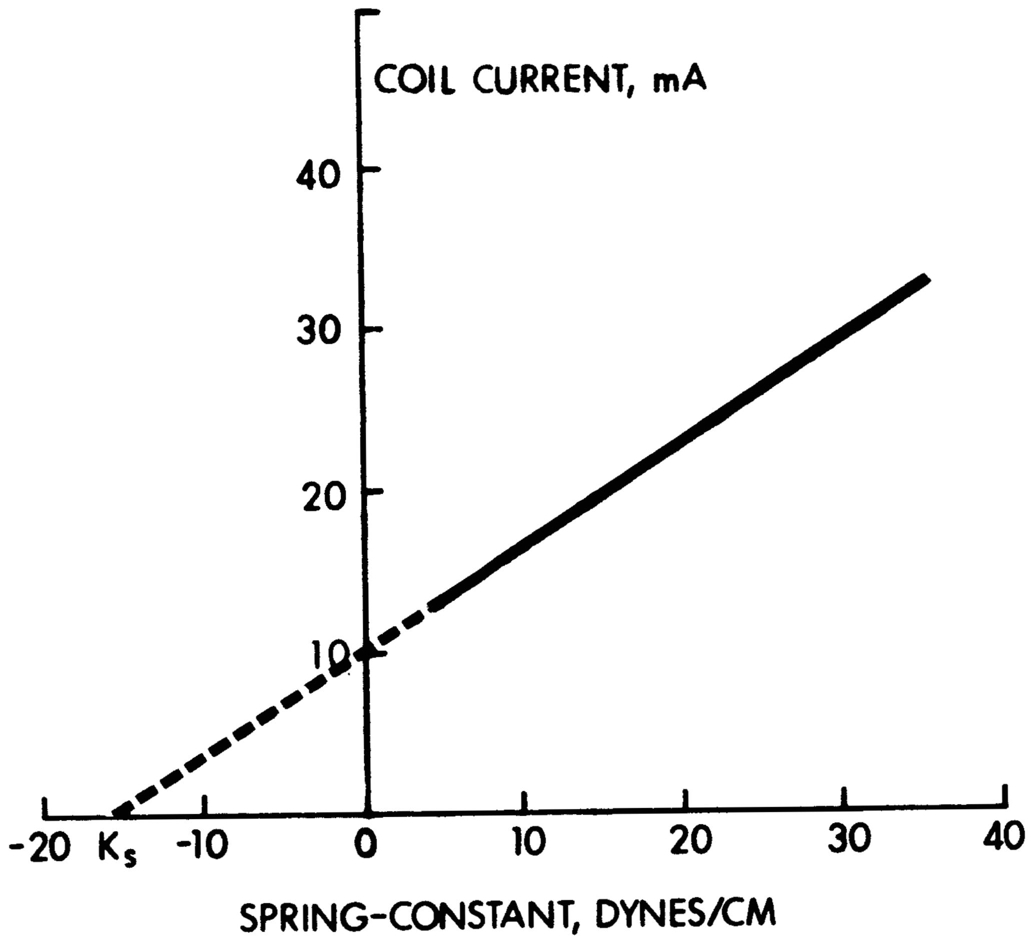

Figure 4 Suspension Axial Spring Constant vs Coil Current

m	=	mass of magnet	τ_c	=	time constant of diff. mode force
k_f	=	suspension damping coefficient	τ_d	=	time constant of displacement detector
k_c	=	gain of coils	I	=	standing current in coils
k_a	=	gain of amplifier	θ	=	tilt error angle (small)
k_d	=	gain of displacement detector			

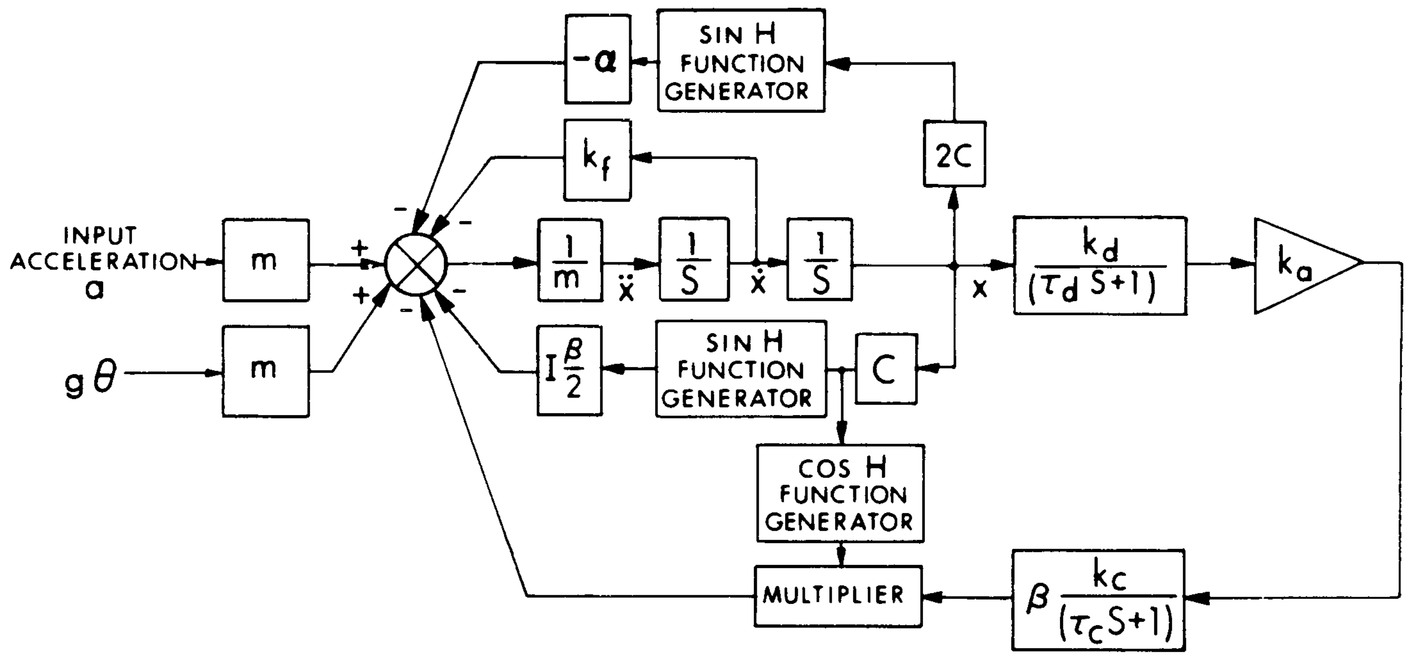

Figure 5 Overall Block Diagram for Accelerometer

about the transverse horizontal axis of the magnet was excited as shown in Figure 6. This excited mode was due to the detector being sensitive to rotation of the magnet and was eliminated by suitable filtering.

The accelerometer in this crude form detected tilts equivalent to 10^{-5}g.

The excitation of a rotary oscillation is at least partly due to the magnetic axis of the test mass not coinciding with its principal axis. This causes the axial restoring force from the coils to apply a torque to the magnet. If the displacement detector is sensitive to rotation, an oscillatory condition can exist.

Figure 7 shows the coupling of the oscillatory mode for a given detector rotational sensitivity α_I and a torque/differential current ratio K. The only place where compensation is feasible is in the servo block $AG_2(s)$, in the main accelerometer loop.

Sensitive Displacement Detector

As mentioned earlier, very small axial displacements, on the order of a fraction of a micron, of the magnet must be detected in order to preserve an adequate bandwidth at low acceleration levels. Another reason for desiring high sensitivity is that high axial spring constant can be tolerated if the displacement is always small.

The interferometric technique was found to offer both high sensitivity and compatibility with the rest of the system. The magnet is easily accessible by the interferometer light beams through the windows on either side of the dewar. The principal mirrors of the interferometer are attached to each end of the magnet as shown in Figure 8.

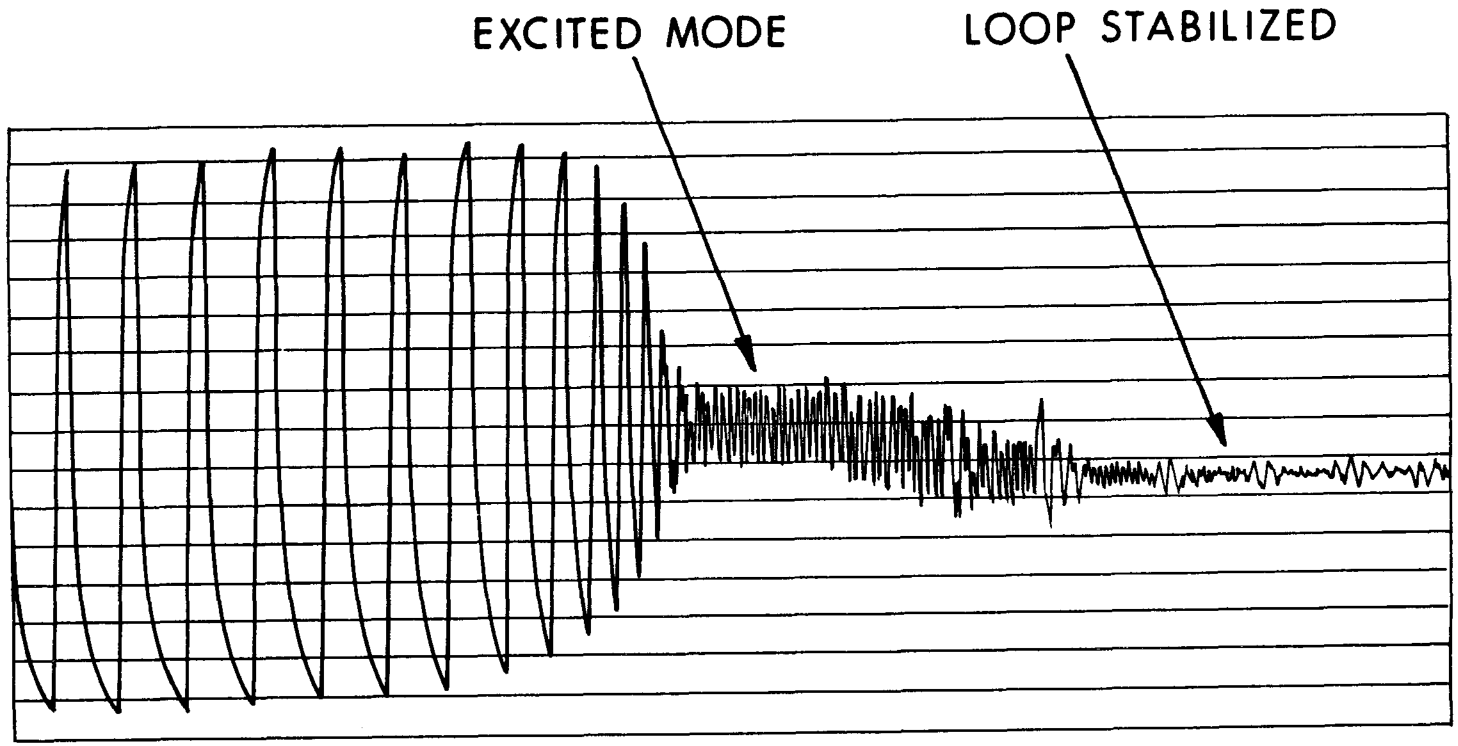

Figure 6 Damping of Closed Loop Oscillation

$G_1(S)$ = AXIAL DYNAMICS OF SUSPENSION
$H_1(S)$ = ROTATIONAL DYNAMICS OF SUSPENSION
$AG_2(S)$ = GAIN AND DYNAMICS OF SERVO
a_1 = DETECTOR AXIAL SENSITIVITY
a_2 = DETECTOR ROTATIONAL SENSITIVITY

γ = CONSTANT: AXIAL FORCE/DIFFERENTIAL CURRENT
K = CONSTANT: TORQUE/DIFFERENTIAL CURRENT
u = NOISE
v = DISTURBANCE FROM ENVIRONMENT

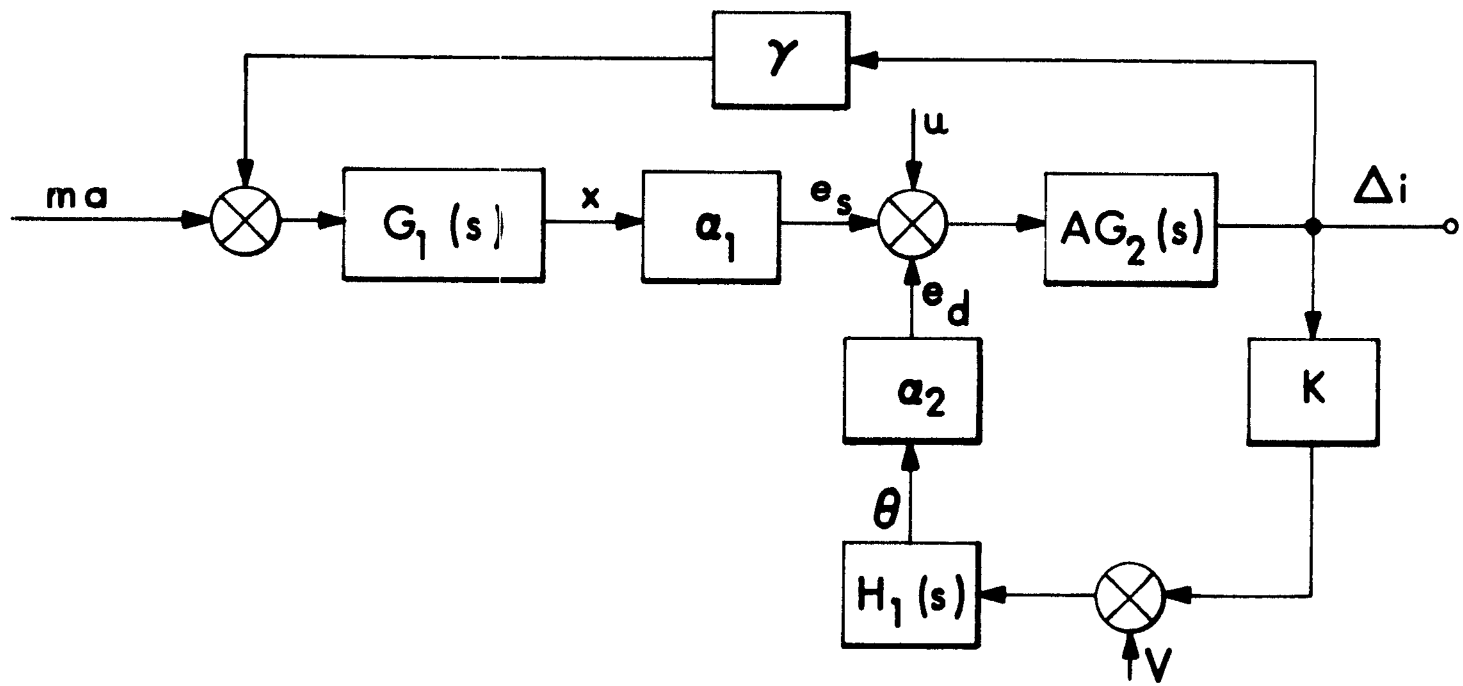

Figure 7 Coupling of Oscillatory Modes

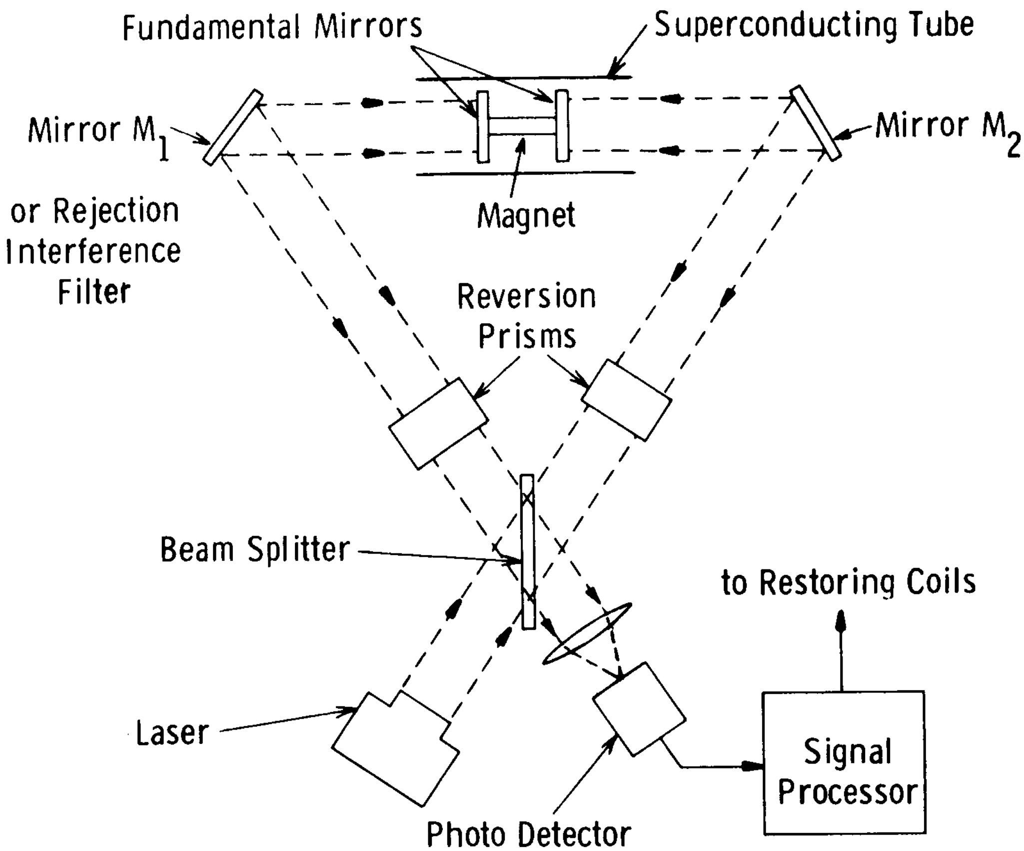

Figure 8 Interferometer Displacement Detector

To obtain more sensitivity and to simplify the method of readout, a Twyman-Green configuration is chosen in which the fringe pattern is a uniform field, modulated in intensity by the axial displacement of the magnet. The advantage of having both principal mirrors on the magnet is that this doubles the sensitivity compared to a normal interferometer and also provides a means for making the displacement detector insensitive to rotation of the magnet about its transverse axes. Another advantage is that both principal mirrors are subjected to the same environmental disturbances.

Normally, when the magnet rotates a little about a transverse axis the interfering beams emerging from the beam splitter no longer coincide and the common overlap area displays the formation of an optical wedge. However, if one principal mirror is optically completely reverted the interfering beams will move together and in addition no wedge will be formed. An equivalent effect can be achieved by placing a three-mirror dove prism, or reversion prism, in each leg of the interferometer, the axes of the prisms being 90° apart.

The use of retro-reflectors (or corner cubes) instead of optical flats at either end of the magnet to eliminate rotation effects is also being investigated. Because of weight and float height limitations, the retro-reflectors have to be small, very light and yet accurate.

The illumination of the interferometer in this configuration must be by a highly collimated beam having a narrow spectral bandwidth. The spatial and temporal coherence of a gas laser beam is very suitable for this application.

After interference, the light emerging from the beam splitter shown in Figure 8 is gathered by a lens and integrated by a photodetector. The output of the photodetector controls the differential current in the restoring coils. The position of the magnet is adjusted so that the output intensity is half way between that corresponding to constructive and destructive interference. Although this output changes with displacement, it does not carry information about the direction of the displacement. This is of no consequence since if the feedback has the wrong sign the magnet will move a distance $\frac{\lambda_d}{2}$, causing a sign reversal, and the magnet will stabilize around this new position. λ_d is one quarter of the wavelength of the light beam and is the distance the magnet moves for the output intensity to vary through a complete cycle.

The Advantages of LLAMA Design

Several advantages of the LLAMA design may be summarized as follows:

a) No mechanism exists to cause threshold-suitable for low level accelerations.

b) Large float height (about 1 cm) - surface roughness or distortion effects are small as compared with an electrostatic suspension.

c) No loss of superconductivity due to external heat input or A. C. losses.

d) Compatibility with interferometric techniques for displacement detection.

e) Compatibility with radiation pressure calibration technique.

Future Work

Experiments are being conducted to verify the feasibility of the interferometer displacement detector and its insensitivity to rotation of the test mass. In order to achieve further increase in bandwidth at low acceleration levels greater displacement sensitivity is required. Frequency rather than phase techniques are being considered, where the mirrors on the magnet form parts of two passive resonant cavities in conjunction with a frequency stabilized gas laser. Using the magnet as part of the laser cavity itself is also being considered.

Several attempts have been made to coat a perfectly smooth copper cylinder with niobium and lead without too much success. Further attempts will be made. Departure from a cylindrical superconducting tube will also be made in the hope of reducing the inherent spring constant of the suspension.

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